Propagation in Transversely Magnetized Compressible Plasma Between Two Parallel Planes

HILLEL UNZ, SENIOR MEMBER, IEEE

Abstract —**The propagation of waves in compressible single fluid macroscopic plasma between two parallel, perfectly conducting planes, with a transverse static magnetic field parallel to the boundaries is investigated. It is shown that the TE waves are not affected by either the static magnetic field or the compressibility of the plasma, while the TM wave will be affected by both.**

I. Introduction

IN RECENT years there has been great interest in gyrotropic media propagation and their various waveguide applications. Most of the theoretical and experimental published works discussed propagation in gyrotropic magnetized ferrites and their applications as waveguide components. Gamo [1], Kales [2], Suhl and Walker [3], and Ginzburg [4] solved the problem of a circular waveguide completely filled with a longitudinally magnetized ferrite. Van Trier [5] discussed the modes which can exist in parallel plane waveguide. Mikaelyan [6] solved the problem of propagation between parallel planes perpendicular to the static magnetic field. Barzilai and Gerosa [7] discussed the different modes in rectangular waveguides filled with magnetized ferrites. A general theoretical approach to this problem was given by Epstein [8]. Numerous other references may be found elsewhere [9], [10].

The similarity between the permeability tensor describing the behavior of the magnetized ferrite and the dielectric tensor describing the behavior of the magneto-ionic plasma has been pointed out previously [3], [8]. Experimental results of propagation of electromagnetic waves in a rectangular waveguide filled with magnetized plasma were given by Goldstein [11].

The aim of the present paper is to solve a waveguide boundary value problem for compressible magnetoplasma, taking into account the electron-gas-dynamics as well as the electromagnetic boundary conditions. The propagation of waves will be discussed in compressible single fluid macroscopic plasma between two perfectly conducting parallel planes, with transverse magnetostatic field parallel to the boundaries.

Manuscript received October 2, 1981; revised January 5, 1982.
The author is with the Department of Electrical Engineering, University of Kansas, Lawrence, KS 66045.

The two parallel perfectly conducting planes are given at $x=0$ and at $x=a$. The transverse static magnetic field parallel to the boundaries is given in the y direction, with the waves propagating in the z direction.

II. The Basic Equations

In the macroscopic equations in the continuum theory of plasma dynamics for single fluid hot compressible magnetoplasma are given by six partial differential equations, three of them vector equations and three of them scalar equations [12]–[14], in terms of the following variables:

$\bar{E}$ electric field in V/m,
$\bar{H}$ magnetic field in A/m,
$\bar{u}$ velocity vector in m/s,
p pressure in kg/ms^2,
N number density in 1/m^3,
T temperature in K.

Denoting the stationary values of the variables of the plasma by subscript 0, it will be assumed that the plasma is stationary ($\bar{u}_0=0$), that there is no net electrostatic field ($\bar{E}_0=0$) and that there is an arbitrarily directed static magnetic field ($\bar{H}_0 \neq 0$). It will be further assumed that the electron gas is an inviscid fluid ($\eta=0$), that there is no heat conduction ($\chi=0$) and that the collision frequency of the electrons with the neutral particles may be neglected ($\nu=0$).

Using "small signal theory" approximation and assuming harmonic time variation $e^{+i\omega t}$, the basic equations for the compressible hot magnetoplasma could be written in the following form, using the above assumptions in the electromagneto-plasma-gas-dynamics equations [12]–[14].

The Maxwell's equations

$$\nabla \times \bar{E} = -i\omega\mu\bar{H} \tag{1a}$$

$$\nabla \times \bar{H} = i\omega\epsilon\bar{E} - eN_0\bar{u}. \tag{1b}$$

The equation of state

$$\frac{p}{p_0} = \frac{N}{N_0} + \frac{T}{T_0}, \qquad p_0 = KN_0T_0. \tag{1c}$$

0018-9480/82/0600-0894$00.75

The conservation of mass

$$N_0\nabla\cdot\bar{u} = -i\omega N. \tag{1d}$$

The conservation of momentum

$$i\omega m N_0\bar{u} = -\nabla p - eN_0\bar{E} - e\mu N_0\bar{u}\times\bar{H}_0. \tag{1e}$$

The conservation of energy

$$i\omega m N_0 C_v T = -p_0\nabla\cdot\bar{u}. \tag{1f}$$

We define

μ permeability of free space,
ϵ dielectric constant of free space,
e positive electron charge,
m mass of electron,
K $m(C_p - C_v) =$ Boltzmann constant,
$\mathbb{S}$ $\frac{C_p}{C_v} = \frac{\text{specific heat in constant pressure}}{\text{specific heat in constant volume}}$.

Equations (1) describe the behavior of the plasma waves in single fluid hot compressible magnetoplasma in terms of three vector equations (1a), (1b), (1e), and three scalar (1c), (1d), (1f), with the stationary known constants $p_0, N_0, T_0, \bar{H}_0$ and the varying unknown wave components $\bar{E}, \bar{H}, \bar{u}, p, N, T$.

From (1c), (1d), and (1f) one finds the following:

$$\frac{N}{N_0} = \frac{1}{\mathbb{S}}\frac{p}{p_0} = \frac{1}{\mathbb{S}-1}\frac{T}{T_0} = \frac{-1}{i\omega}\nabla\cdot\bar{u}. \tag{2}$$

Substituting (1a) into (1b)

$$\nabla\times\nabla\times\bar{E} - k_0^2\bar{E} = i\omega\mu e N_0\bar{u} \tag{3a}$$

where

$$k_0 = \omega\sqrt{\mu\epsilon} = \frac{\omega}{c} = \text{free-space wavenumber}.$$

Substituting (2) into (1(e))

$$i\omega m N_0\bar{u} = \frac{\mathbb{S}p_0}{i\omega}\nabla(\nabla\cdot\bar{u}) - eN_0\bar{E} - e\mu N_0\bar{u}\times\bar{H}_0. \tag{3b}$$

Let us define

$$a_1 = \sqrt{\frac{\mathbb{S}p_0}{mN_0}} = \sqrt{\mathbb{S}T_0(C_p - C_v)}$$

$$= \text{acoustic velocity in electron gas.}$$

$$k_1 = \frac{\omega}{a_1} = \frac{c}{a_1}k_0 = \text{acoustic wavenumber}$$

$$\bar{Y} = \frac{e\mu\bar{H}_0}{m\omega}.$$

Dividing (3b) by $i\omega m N_0$ on both sides and using the above definitions

$$\bar{u} = -\frac{1}{k_1^2}\nabla(\nabla\cdot\bar{u}) + \frac{ie}{\omega m}\bar{E} + \bar{u}\times i\bar{Y} \tag{3c}$$

and substituting (3a) into (3c) and defining $X = e^2N_0/\omega^2\epsilon m$

$$-\frac{1}{k_0^2}\nabla\times\nabla\times\bar{E} + \frac{1}{k_1^2}\nabla(\nabla\cdot\bar{E}) + (1-X)\bar{E} + \frac{1}{k_0^2}(\nabla\times\nabla\times\bar{E} - k_0^2\bar{E})\times i\bar{Y} = 0. \tag{4}$$

Equation (4) represents the wave equation for the electric field E.

Once the electric field is found using (4), one may find the magnetic field $\bar{H}$ from (1a):

$$\bar{H} = \frac{-1}{i\omega\mu}\nabla\times\bar{E} \tag{5a}$$

and the velocity field $\bar{u}$ from (3a)

$$\bar{u} = \frac{i\omega\epsilon}{eN_0}\left(\bar{E} - \frac{1}{k_0^2}\nabla\times\nabla\times\bar{E}\right). \tag{5b}$$

The rest of the variables may be found by substituting (5b) into (2)

$$\frac{N}{N_0} = \frac{1}{\mathbb{S}}\frac{p}{p_0} = \frac{1}{\mathbb{S}-1}\frac{T}{T_0} = -\frac{\epsilon}{eN_0}\nabla\cdot\bar{E}. \tag{5c}$$

By assuming one arbitrary component of the electric field $\bar{E}$, for example E_z, to be given in the rectangular system of coordinates, one may find all the rest of the plasma wave components, $E_x, E_y, \bar{H}, \bar{u}, p, N, T$ in terms of E_z using (4) and (5).

III. Propagation in Parallel Plane Waveguide

Let us assume that the compressible magnetoplasma is confined by two perfectly conducting parallel planes at $x=0$ and $x=a$ with the magnetostatic field in the y direction:

$$\bar{Y} = Y\hat{y} = \frac{e\mu H_0}{m\omega}\hat{y}. \tag{6}$$

Since the solution will be independent of y, one may assume that each one of the plasma wave components will be in the form

$$E_j(x,z) = E^j(\alpha)e^{i\alpha x}e^{i(\omega t - \gamma z)} \tag{7}$$

where $j = x, y, z$.

The constant γ will represent the propagation constant of the plasma wave modes in the z direction; it will be dependent on α which will be determined from the boundary conditions.

Substituting (6) and (7) into (4) and taking from (7) $\partial/\partial x = i\alpha$, $\partial/\partial y = 0$, $\partial/\partial z = -i\gamma$:

$$\begin{bmatrix} G_{11} & 0 & G_{13} \\ 0 & G_{22} & 0 \\ G_{31} & 0 & G_{33} \end{bmatrix}\begin{bmatrix} E^x \\ E^y \\ E^z \end{bmatrix} = 0 \tag{8a}$$

$$G_{11}=k_0^2(1-X)-\sigma\alpha^2-\gamma^2-i\alpha\gamma Y \tag{8b}$$
$$G_{13}=-(1-\sigma)\alpha\gamma+i(k_0^2-\alpha^2)Y \tag{8c}$$
$$G_{22}=k_0^2(1-X)-\alpha^2-\gamma^2 \tag{8d}$$
$$G_{31}=-(1-\sigma)\alpha\gamma-i(k_0^2-\gamma^2)Y \tag{8e}$$
$$G_{33}=k_0^2(1-X)-\alpha^2-\sigma\gamma^2+i\alpha\gamma Y \tag{8f}$$

where $k_0^2=\omega^2\mu\epsilon$ and $\sigma=a_1^2/c^2=k_0^2/k_1^2$.

For a nontrivial solution the determinant of the coefficients (8) should be zero. Developing the determinant

$$G_{22}=k_0^2(1-X)-\alpha^2-\gamma^2=0 \tag{9a}$$

$$G_{11}G_{33}-G_{13}G_{31}=\sigma(\alpha^2+\gamma^2)^2-k_0^2(\alpha^2+\gamma^2)\cdot\left[(1+\sigma)(1-X)-Y^2\right]+k_0^4\left[(1-X)^2-Y^2\right]=0. \tag{9b}$$

It will be shown later that (9a) represents the TE mode while (9b) represents the TM mode.

According to the theory of linear equations, one may disregard now one of the equations in (8a), for example the last one, and obtain from the two remaining equations:

$$G_{22}E^Y=0 \tag{10a}$$

$$E^x=-\frac{G_{13}}{G_{11}}E^z=-\frac{G_{33}}{G_{31}}E^z. \tag{10b}$$

The rest of the plasma wave components will be found by using (5).

The following boundary conditions will be applied to the present problem:

$$E_z=0 \qquad x=0,a \tag{11a}$$
$$E_y=0 \qquad x=0,a \tag{11b}$$
$$u_x=0 \qquad x=0,a. \tag{12}$$

IV. The TE Modes

In the present section we will discuss the TE mode, where there is no electric field component in the direction of propagation z, one derives from (10)

$$E_z=E_x=0. \tag{13a}$$

Using (7) and the boundary condition (11b)

$$E_y=A\sin\frac{m\pi}{a}xe^{i(\omega t-\gamma z)} \tag{13b}$$

where $m=$ integer. From (9a) the propagation constant is

$$\gamma^2=k_0^2(1-X)-\left(\frac{m\pi}{a}\right)^2. \tag{14}$$

Using (13) and (5a) one obtains

$$H_x=\frac{1}{i\omega\mu}\frac{\partial Ey}{\partial z}=-\frac{\gamma}{\omega\mu A}\sin\frac{m\pi}{a}xe^{i(\omega t-\gamma z)} \tag{15a}$$

$$H_y=0 \tag{15b}$$

$$H_z=-\frac{1}{i\omega\mu}\frac{\partial Ey}{\partial x}=+\frac{i}{\omega\mu}\frac{m\pi}{a}A\cos\frac{m\pi}{a}xe^{i(\omega t-\gamma z)}. \tag{15c}$$

From (5b), (13), and (14)

$$u_x=u_z=0 \tag{16a}$$

$$u_y=\frac{ie}{\omega m}E_y=\frac{ie}{\omega m}A\sin\frac{m\Pi}{a}xe^{i(\omega t-\gamma z)} \tag{16b}$$

From (5c) and (13) is

$$N=p=T=0. \tag{17}$$

Equations (13)–(17) give the plasma wave components and the propagation constant of the TE wave. The TE wave is not influenced at all by the static magnetic field since the motion of the plasma is along the magnetostatic field. The TE wave is not affected by the compressibility of the plasma.

V. The TM Modes

The propagation constant equation for the TM modes is given by (9b). Assuming that the propagation constant γ is given, one may solve the quadratic equation (9b)

$$\frac{2\sigma}{k_0^2}(\alpha_{1,2}^2+\gamma^2)=(1+\sigma)(1-X)-Y^2\pm\left\{\left[(1-\sigma)(1-X)-Y^2\right]^2+4\sigma XY^2\right\}^{1/2}. \tag{18}$$

For each propagation constant $\pm\gamma$ one has four related solutions $\pm\alpha_1$ and $\pm\alpha_2$.

Using (10) and (18) one has

$$E_y=0 \tag{19a}$$

$$E_z=\left[A_1\sin\alpha_1x+B_1\cos\alpha_1x+A_2\sin\alpha_2x+B_2\cos\alpha_2x\right]e^{i(\omega t-\gamma z)} \tag{19b}$$

where α_1,α_2 are related to γ in accordance with (18).

From (10b) one obtains

$$\frac{E^x}{E^z}=-\frac{G_{33}}{G_{31}}=iC(\alpha^2)+D(\alpha^2)\alpha \tag{20a}$$

where

$$C(\alpha^2)=Y\frac{(1-\sigma)\alpha^2\gamma^2-(k_0^2-\gamma^2)\left[k_0^2(1-X)-\alpha^2-\sigma\gamma^2\right]}{(1-\sigma)^2\alpha^2\gamma^2+(k_0^2-\gamma^2)^2Y^2}$$

$$D(\alpha^2)=\frac{(1-\sigma)\gamma\left[k_0^2(1-X)-\alpha^2-\sigma\gamma^2\right]+\gamma Y^2(k_0^2-\gamma^2)}{(1-\sigma)^2\alpha^2\gamma^2+(k_0^2-\gamma^2)^2Y^2}.$$

Using the operator $\partial/\partial x=+i\alpha$, (20a) may be rewritten as follows:

$$E_x = i\left[C(\alpha^2)E_z - D(\alpha^2)\frac{\partial E_z}{\partial x}\right]. \tag{20b}$$

Using the notation $C(\alpha_{1,2}^2) = C_{1,2}$ and $D(\alpha_{1,2}^2) = D_{1,2}$ one may obtain by substituting (19b) into (20b):

$$E_x = i[P_1 \sin\alpha_1 x + Q_1 \cos\alpha_1 x + P_2 \sin\alpha_2 x + Q_2 \cos\alpha_2 x]e^{i(\omega t - \gamma z)} \tag{19c}$$

where we define

$$\begin{aligned} P_1 &= A_1C_1 + \alpha_1 B_1 D_1 \\ Q_1 &= B_1C_1 - \alpha_1 A_1 D_1 \\ P_2 &= A_2C_2 + \alpha_2 B_2 D_2 \\ Q_2 &= B_2C_2 - \alpha_2 A_2 D_2. \end{aligned}$$

By substituting (19) into (5a) one may find the magnetic field as follows:

$$H_x = H_z = 0 \tag{21a}$$

$$H_y = \frac{i}{\omega\mu}\left[\frac{\partial}{\partial x}E_z - i\gamma E_x\right] \tag{21b}$$

and substituting (19) into (21b)

$$H_y = \frac{i}{\omega\mu}[R_1 \sin\alpha_1 x + S_1 \cos\alpha_1 x + R_2 \sin\alpha_2 x + S_2 \cos\alpha_2 x]e^{i(\omega t - \gamma z)} \tag{21c}$$

where we define

$$\begin{aligned} R_1 &= -B_1\alpha_1 + \gamma P_1 \\ S_1 &= A_1\alpha_1 + \gamma Q_1 \\ R_2 &= -B_2\alpha_2 + \gamma P_2 \\ S_2 &= A_2\alpha_2 + \gamma Q_2. \end{aligned}$$

Since $H_z = 0$ in (21a) we are justified in calling it the TM mode of propagation.

By substituting (19) into (5b) one may find

$$u_y = 0 \tag{22a}$$

$$u_x = \frac{i\omega\epsilon}{eN_0k_0^2}\left[(k_0^2 - \gamma^2)E_x + i\gamma\frac{\partial E_z}{\partial x}\right] \tag{22b}$$

$$u_z = \frac{i\omega\epsilon}{eN_0k_0^2}\left[(k_0^2 - \alpha^2)E_z + i\gamma\frac{\partial E_x}{\partial x}\right]. \tag{22c}$$

Substituting (19) into (5c)

$$\frac{N}{N_0} = \frac{1}{\mathfrak{S}}\frac{p}{p_0} = \frac{1}{\mathfrak{S}-1}\frac{T}{T_0} = \frac{\epsilon}{eN_0}\left[i\gamma E_z - \frac{\partial E_x}{\partial x}\right] \tag{22d}$$

and (19b) and (19c) into (22b) becomes

$$u_x = -\frac{\omega\epsilon}{eN_0k_0^2}[V_1 \sin\alpha_1 x + W_1 \cos\alpha_1 x + V_2 \sin\alpha_2 x + W_2 \cos\alpha_2 x]e^{i(\omega t - \gamma z)} \tag{23}$$

where one has

$$\begin{aligned} V_1 &= (k_0^2 - \gamma^2)P_1 - \gamma\alpha_1 B_1 \\ W_1 &= (k_0^2 - \gamma^2)Q_1 + \gamma\alpha_1 A_1 \\ V_2 &= (k_0^2 - \gamma^2)P_2 - \gamma\alpha_2 B_2 \\ W_2 &= (k_0^2 - \gamma^2)Q_2 + \gamma\alpha_2 A_2. \end{aligned} \tag{24}$$

Substituting the definitions from (19c) one could rewrite the above as follows:

$$\begin{aligned} V_1 &= G_1A_1 + H_1B_1 \\ W_1 &= -H_1A_1 + G_1B_1 \\ V_2 &= G_2A_2 + H_2B_2 \\ W_2 &= -H_2A_2 + G_2B_2 \end{aligned} \tag{25}$$

where we define

$$\begin{aligned} G_1 &= (k_0^2 - \gamma^2)C_1 \\ H_1 &= \alpha_1 D_1(k_0^2 - \gamma^2) - \alpha_1\gamma \\ G_2 &= (k_0^2 - \gamma^2)C_2 \\ H_2 &= \alpha_2 D_2(k_0^2 - \gamma^2) - \alpha_2\gamma. \end{aligned} \tag{26}$$

Similarly one could derive the explicit expressions for u_z, N, p, T by substituting (19) in (22).

In order to evaluate the plasma wave TM mode components one will have to find the characteristic values $\alpha_1, \alpha_2, \gamma$ from the boundary conditions. In the next section we will derive the equation for those eigenvalues.

VI. The TM Modes Characteristic Values

In the present section we will derive the transcendental equation for the characteristic values of the TM modes, subject to the boundary conditions.

Using the boundary conditions (11a) in (19b)

$$B_1 + B_2 = 0 \tag{27a}$$

$$A_1 \sin\alpha_1 a + B_1 \cos\alpha_1 a + A_2 \sin\alpha_2 a + B_2 \cos\alpha_2 a = 0. \tag{27b}$$

From (19a) one sees that the boundary conditions (11b) are satisfied identically.

Substituting the boundary conditions (12) in (23) and using (25)

$$(-H_1A_1 + G_1B_1) + (-H_2A_2 + G_2B_2) = 0 \tag{28a}$$

$$(G_1A_1 + H_1B_1)\sin\alpha_1 a + (-H_1A_1 + G_1B_1)\cos\alpha_1 a + (G_2A_2 + H_2B_2)\sin\alpha_2 a + (-H_2A_2 + G_2B_2)\cos\alpha_2 a = 0. \tag{28b}$$

From (27) and (28) one obtains four linear homogeneous equations with four unknowns A_1, B_1, A_2, B_2. For a nontrivial solution the determinant of the coefficients should be zero

$$\begin{vmatrix} \underline{A_1} & \underline{B_1} & \underline{A_2} & \underline{B_2} \\ 0 & 1 & 0 & 1 \\ \sin\alpha_1 a & \cos\alpha_1 a & \sin\alpha_2 a & \cos\alpha_2 a \\ -H_1 & +G_1 & -H_2 & +G_2 \\ G_1\sin\alpha_1 a- & H_1\sin\alpha_1 a+ & G_2\sin\alpha_2 a- & H_2\sin\alpha_2 a+ \\ -H_1\cos\alpha_1 a & +G_1\cos\alpha_1 a & -H_2\cos\alpha_2 a & +G_2\cos\alpha_2 a \end{vmatrix} = 0 \tag{29}$$

where $G_1(\gamma,\alpha_1)$, $H_1(\gamma,\alpha_1)$, $G_2(\gamma,\alpha_2)$, $H_2(\gamma,\alpha_2)$ are defined by (26). Developing the determinant (29)

$$2H_1H_2(1-\cos\alpha_1 a\cos\alpha_2 a) = \left[H_1^2+H_2^2+(G_1-G_2)^2\right]\sin\alpha_1 a\sin\alpha_2 a. \tag{30}$$

Substituting the values of $|\alpha_1|$ and $|\alpha_2|$ from (18) expressed in terms of γ and the plasma parameters in (30) one obtains a transcendental equation for the propagation constant γ. The solution of this transcendental equation will give an infinite number of discrete solutions for $\gamma=\gamma_m$. For each γ_m of a particular TM mode one may find from (18) α_{1m} and α_{2m} and using those characteristic values one will be able to find the components of the plasma wave of the particular TM_m mode under consideration.

For the particular case of incompressible cold plasma one has $a_1=0$ and $\sigma=0$ and (9b) becomes

$$\alpha^2+\gamma^2=k_0^2\frac{(1-X)^2-Y^2}{1-X-Y^2}. \tag{31}$$

From (31) one has $\alpha_1=\alpha_2=\alpha$ and substituting it in (30)

$$\left[(H_1-H_2)^2+(G_1-G_2)^2\right]\sin^2\alpha a=0 \tag{32}$$

where (32) is an identity since

$$H_1(\alpha_1)=H_2(\alpha_2)$$

and

$$G_1(\alpha_1)=G_2(\alpha_2).$$

Using the boundary conditions one obtains $\alpha=n\pi/a$.

Once the determinant of the coefficients (27), (28) has been satisfied through (30), one may ignore one of the equations, for example (28b) and rewrite (27) and (28) in the form

$$B_1+B_2=0 \tag{33a}$$

$$B_1\cos\alpha_1 a+B_2\cos\alpha_2 a+A_2\sin\alpha_2 a=-A_1\sin\alpha_1 a \tag{33b}$$

$$G_1B_1+G_2B_2-H_2A_2=H_1A_1. \tag{33c}$$

From (33) one may find

$$\frac{A_2}{A_1}=\frac{H_1(\cos\alpha_2 a-\cos\alpha_1 a)+(G_2-G_1)\sin\alpha_1 a}{H_2(\cos\alpha_1 a-\cos\alpha_2 a)-(G_2-G_1)\sin\alpha_2 a} \tag{34a}$$

$$\frac{B_1}{A_1}=\frac{H_1\sin\alpha_2 a-H_2\sin\alpha_1 a}{H_2(\cos\alpha_1 a-\cos\alpha_2 a)-(G_2-G_1)\sin\alpha_2 a} \tag{34b}$$

and

$$\frac{B_2}{A_1}=\frac{H_2\sin\alpha_1 a-H_1\sin\alpha_2 a}{H_2(\cos\alpha_1 a-\cos\alpha_2 a)-(G_2-G_1)\sin\alpha_2 a}=-\frac{B_1}{A_1}. \tag{34c}$$

For an arbitrary value of A_1, one may find A_2, B_1, B_2, from (34). Substituting those values in (19b) one may find E_z in terms of the arbitrary amplitude A_1. Once E_z is known, the rest of the plasma TM mode wave components may be found by using the relations in the previous section.

VII. Summary

The propagation of plasma waves in compressible single fluid macroscopic plasma, between two parallel perfectly conducting planes with transverse magnetostatic field parallel to the boundaries, has been discussed. Both the electromagnetic and the electron-gas-dynamics boundary conditions have been taken into account. It has been shown that the TE plasma mode of propagation will not be affected by either the magnetostatic field or by the compressibility of the plasma. The TM plasma mode of propagation will be affected by both the static magnetic field and the compressibility of the plasma.

Other cases of propagation between parallel plane waveguides with magnetized compressible plasma will be discussed elsewhere.

References

[1] H Gamo, "Faraday rotation of waves in a circular waveguide," *Phys Soc. Japan J.*, vol 8, pp 176–182, 1953

[2] M L. Kales, "Modes in waveguides containing ferrites," *J Appl. Phys.*, vol 24, pp 604–608, 1953.

[3] H Suhl and L R Walker, "Topics in guided wave propagation through gyromagnetic media," *Bell Syst. Tech. J.*, vol. 33, pp. 579–659, 939–986, 1133–1194, 1954.

[4] A. M. Ginzburg, "Gyrotropic waveguides," *Doklady Akad. Nauk. USSR*, vol. 95, pp. 489–492, 1954.

[5] A. A. Th. M. Van Trier, "Guided electromagnetic waves in anisotropic media," *Appl. Sci. Res*, vol B-3, pp. 305–371, 1953

[6] A. L. Mikaelyan, "Electromagnetic waves in rectangular waveguide filled with magnetized ferrite," *Doklady Akad. Nauk USSR*, vol 98, pp. 941–944, 1954.

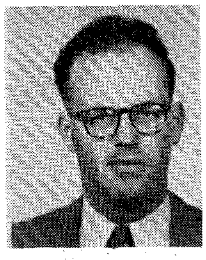
[7] G. Barzilai and G. Gerosa, "Modes in rectangular guides filled with magnetized ferrite," *Il Nuovo Cimento*, vol X-7, pp. 685–697, 1958

[8] P. S Epstein, "Theory of wave propagation in a gyromagnetic medium," *Rev. Mod. Phys.*, vol. 28, pp 3–17, 1956.

[9] B. Lax and K. J Button, *Microwave Ferrites and Ferrimagnetics* New York: McGraw Hill, 1962

[10] H. Unz, "Propagation in arbitrarily magnetized ferrites between two conducting parallel planes," *IEEE Trans Microwave Theory Tech.*, vol MTT-11, pp 204–210, 1963.

[11] L. Goldstein, "Nonreciprocal electromagnetic wave propagation in ionized gaseous media," *IRE Trans., Microwave Theory Tech.*, vol. MTT-6, pp. 19–29, 1958.
[12] S. I. Pai, *Magnetogasdynamics and Plasma Dynamics.* Englewood Cliffs, N.J.: Prentice Hall, 1962.
[13] H. Unz, "Wave propagation in drifting isotropic warm plasma," *Radio Sci.*, vol. 1, pp. 325–338, 1966.
[14] H. Unz, "Energy transfer in macroscopic plasma," *Radio Sci.*, vol. 12, pp. 993–999, 1977.



Hillel Unz (S'53–M'57–SM'60) was born in Darmstadt, Germany, on August 15, 1929. He went to Israel in 1931 and served there in the Israel Defense Forces during the war of independence 1947–1949. He received the B.S. in electronics, summa cum laude, in 1953 and the Diplome Engineer degree in 1961, both from Israel Institute of Technology (Technion) Haifa, Israel. He received the M.S. degree and the Ph.D. degree from the University of California, Berkeley in 1954 and 1957, respectively.

From 1953 until 1957, he was a Research Assistant in the Microwave Antenna Laboratory, Electrical Engineering Department, University of California, Berkeley. Since February 1957, he has been on the staff of the Electrical Engineering Department, University of Kansas, Lawrence, first as an Assistant Professor, and since 1962, as a full Professor of Electrical Engineering. He was a consultant for the Radiation Laboratory and the Infrared Laboratory, University of Michigan, Ann Arbor, Research Associate in the Ionosphere Research Laboratory, Pennsylvania State University, University Park, and a Research Associate in the Antenna Laboratory, Ohio State University, Columbus. He was also a consultant for Midwest Research Institute, Kansas City, MO, for Texas Instruments, Inc., Dallas, for the Antenna Laboratory, Ohio State University, Columbus, and to the RAND Corporation, Santa Monica, CA. During 1963–1964 and 1970–1971 he was on sabbatical leave from the University of Kansas. He was a National Science Foundation Post-Doctoral Fellow in the Cavendish Laboratory, Cambridge University, England and UNESCO Specialist in Applied Mathematics under the Expanded Technical Assistance Program in the Mathematics Department Israel Institute of Technology (Technion), Haifa, Israel. He has done research work in electromagnetic theory and boundary value problems, antenna arrays and electronic scanning, wave propagation in the ionosphere and in moving magnetized plasma, electromagneto-plasma-gas-dynamics, interaction between electromagnetic waves–acoustic waves, interaction between acoustic waves–clamped mechanical plates, and others. Dr. Unz has published more than 100 scientific reviewed publications and about 40 scientific reports. He is the co-author of a book (research monograph) titled, *Electromagnetic Waves in Moving Magneto-Plasmas*. One of his outstanding contributions is his invention of the nonuniformly spaced array and the development of its basic theory, and he has been granted four U. S. Patents on this subject.

Network Representation and Transverse Resonance for Layered Anisotropic Dielectric Waveguides

OTTO SCHWELB, MEMBER, IEEE

***Abstract*—First, the matrix wave impedance in an unbounded uniaxial lossless dielectric material is determined. Next, the transformation properties of the input impedance of a terminated anisotropic layer are established. It is then demonstrated that the boundary conditions in an anisotropic dielectric slab waveguide lead to a generalized transverse resonance condition involving the previously obtained matrix input impedances. Network equivalent representations are given for waveguides fabricated with dielectrics in polar and longitudinal orientations. The results show that a circuit approach to the analysis and design of planar anisotropic dielectric waveguides is feasible and practicable.**

Manuscript received November 27, 1981; revised February 1, 1982. This work was supported by the Natural Sciences and Engineering Research Council, Canada.
The author is with the Department of Electrical Engineering, Concordia University, Montreal, Que., Canada H3G 1M8.

I. INTRODUCTION

THE CONCEPT of impedance and equivalent network representation is often used to obtain the dispersion characteristics of isotropic waveguides. As a result of the additional coupling mechanisms acting between field components in an anisotropic dielectric, the wave impedance expands into matrix form, and circuit equivalents are a great deal more cumbersome than those in the isotropic case. For this reason anisotropic layered waveguides are seldom treated by the methods of circuit analysis. Yet, there are some important configurations where the network approach provides both insight and a simple solution to the guidance problem.

0018-9480/82/0600-0899$00.75 ©1982 IEEE